Math 198 **Name:**

Problem 1.

Give an example of a graph $G = (V, E)$ with cycle space $\mathcal{C}$ for which

$$\dim \mathcal{C} = \#(E) - \#(V) + 1,$$

and give an example of another graph for which that is not the case. In what types of graph does this equality hold? Why?

Problem 2.

Recall that given a vector space V over field $\mathbb{F}$, its dual is defined to be $V^* := \mathrm{Hom}_{\mathbb{F}}(V, \mathbb{F})$. Recall further that given another vector space W over $\mathbb{F}$ and a linear transformation $T \in \mathrm{Hom}_{\mathbb{F}}(V, W)$, there is a natural induced map $T^* : W^* \to V^*$ such that $T^*(g) = g \circ T$ for all $g \in W^*$.

Now consider the **double dual** of a vector space, $V^{**} := \mathrm{Hom}_{\mathbb{F}}(V^*, \mathbb{F})$. Given a map $T : V \to W$ (i.e., $T \in \mathrm{Hom}_{\mathbb{F}}(V, W)$), suggest a definition for its double dual, T^{**}, and decide whether it should go from $V^{**} \to W^{**}$ or from $W^{**} \to V^{**}$. (Make sure to draw diagrams!)

Problem 3.

If M is a finite matroid, we can define the **dual matroid** M^* by taking the same underlying set and calling a set a basis in M^* if and only if its complement is a basis in M.

Verify that M^* is a matroid and that the dual of M^* is M.

(The dual can be described equally well in terms of other ways to define a matroid. For instance: A set is independent in M* if and only if its complement spans M.)